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# WHAT DOES SOCIAL SEMIOTICS HAVE TO OFFER MATHEMATICS EDUCATION RESEARCH?



ABSTRACT. Social Semiotics, based on the work of the linguist Michael Halliday, emphasises the ways in which language functions in our construction and representation of our experience and of our social identities and relationships. In this paper, I provide an introduction to the theory and its analytic tools, considering how they can be applied in the field of mathematics education. Some research questions that may be raised and addressed from this perspective are identified. An illustrative example is offered, demonstrating a social semiotic approach to addressing questions related to construction of the nature of school mathematical activity in writing produced by secondary school students.

KEY WORDS: Halliday, language, linguistics, methodology, nature of mathematics, social semiotics

## 1. Introduction

In recent years, mathematics education research has paid increased attention to social and linguistic context and to the importance of language as the principal medium in which teaching and learning takes place. The 'turn to language' in the theoretical perspectives adopted by researchers in mathematics education has brought with it increased attention to the nature of language and other semiotic systems used in mathematical activity and to the roles that these may play in the teaching, learning and doing of mathematics, drawing on semiotic and linguistic theories and developing them to suit the needs of researchers in this field (see Anderson et al., 2003; Duval, 2000; Sfard, 2000; and articles in this special issue). At the same time, increased numbers of empirical studies have focused on discursive activity within classrooms, especially on interaction between teachers and students (examples may be seen in Cobb et al., 2000; in the special issue of ESM edited by Kieran et al., 2001; and in Steinbring et al., 1998).

My primary concern in this paper, however, is not so much to present an analysis of mathematical language, either in general or in a particular instance, as to discuss the way in which language may serve as a crucial window for researchers on to the processes of teaching, learning and doing mathematics, where these are conceived of as socially organised, that is, not only taking place within a social environment but structured by that

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environment. I shall argue that Halliday's theory of language as social semiotic (Halliday, 1978; Halliday and Hasan, 1989) and the associated tools of systemic functional linguistics (Halliday, 1985) provide some powerful ways of investigating mathematical practices and the practices of teaching and learning mathematics, as well as allowing us to develop knowledge about uses of language within mathematical practices that may be helpful for teaching and learning.

An important starting point for a social semiotic perspective is the recognition that meaning making occurs in social contexts and that language use is functional within those contexts. The context in which language is used and in which learning takes place has become a prominent theme of recent developments in theories of discourse as well as in theories of learning, with many researchers drawing on Vygotskian perspectives on learning. The construction of mathematical meanings has come to be seen as occurring in interaction between teacher and students or among students and insights into processes of construction have been provided by analyses of such interaction using various approaches to discourse and semiotics (Kieran, 2001; Radford, 2000; Sáenz-Ludlow, 2004; Sfard, 2001). The focus of such studies has, in the main, been cognitive, concerned primarily with tracking the development of mathematical concepts through the interaction of the participants, though studies of approaches to mathematical argumentation (Zack and Graves, 2001), competence in participation in classroom discussion (Forster and Taylor, 2003) and different patterns of attention in working with word problems (Barwell, 2003) have addressed a wider range of discursive functions. An important contribution of social semiotics is its recognition of the range of functions performed by use of language and other semiotic resources. Every instance of mathematical communication is thus conceived to involve not only signification of mathematical concepts and relationships but also interpersonal meanings, attitudes and beliefs. This allows us to address a wide range of issues of interest to mathematics education and helps us to avoid dealing with cognition in isolation from other aspects of human activity.

A further contribution of social semiotics is in its conceptualisation of 'context'. The nature of 'context' as it is operationalised in the types of studies mentioned above tends to be restricted to the immediate context of the particular classroom or even the particular episode of activity being studied.[1] From a social semiotic perspective, 'context' is broader than this, incorporating consideration of the culture outside the classroom, as will be discussed below. This conceptualisation is compatible with approaches to discourse that draw on Foucauldian perspectives, perhaps most familiar to mathematics educators from the work of Walkerdine (1988, 1989), providing in addition an associated linguistic theory allowing the detailed analysis of texts situated in their contexts, such as those produced by Critical Discourse Analysis (Chouliaraki and Fairclough, 1999; Fairclough, 1995).

## 2. Language as social semiotic

At its most basic level, a social semiotic perspective involves recognising that language consists of "the exchange of meanings in interpersonal contexts of one kind or another" (Halliday, 1978, p. 2) and that this exchange of meanings is functional. Individuals do not speak or write simply to externalise their personal understandings but to achieve effects in their social world. Studying language and its use must thus take into account both the immediate situation in which meanings are being exchanged (the context of situation) and the broader culture within which the participants are embedded (the context of culture).[2] The *context of situation* encompasses the goals of the current activity, the other participants, the tools available and other aspects of the immediate environment. Each situation cannot be considered in isolation but as an example of a situation type or semiotic structure formed out of the sociosemiotic variables: field, tenor and mode. The *field* of discourse may be thought of not simply as the subject matter but as the institutional setting of the activity in which a speaker and other participants are engaged. *Tenor* encompasses the relationships between the participants, and *mode* refers to the channel of communication (e.g., writing or speech) and other aspects of the role of language in the situation. Within mathematics education, Atweh et al. (1998) have used the structure of the first two of these sociosemiotic variables to analyse mathematics classrooms, identifying differences in both aspects in interactions between teachers and students, apparently related to gender and perceived socio-economic class.

The context of culture includes broader goals, values, history and organising concepts that the participants hold in common. This formulation of *context of culture* suggests a uniformity of culture both between and within the participants. As will become apparent later in this paper, assuming such uniformity is not justified and the notion of participation in multiple discourses will be used as an alternative way of conceptualising this level of context. Importantly, however, the thinking and meaning making of individuals is not simply set within a social context but actually arises through social involvement in exchanging meanings. This dialectical and dynamic conception of the relationship between the individual and the social is compatible, Hodge and Kress (1988) argue, with the theories of Volosinov and Vygotsky.

To illustrate the importance of taking both these aspects of context into account, I shall briefly consider part of the analysis of an extract of data presented in a recent paper addressing the issue of emotion in the mathematics classroom (Morgan et al., 2002a).[3] A group of three boys in a Portuguese middle school classroom were engaged in attempting to find a solution for a mathematical problem. My colleagues and I were interested in identifying possible sources or spaces for emotional experience during the course of the boys' working together on the problem. Considering the context of situation, the field of discourse encompassed the problem itself, the mathematical resources available to the students, and the goal of achieving an acceptable solution; the tenor included relationships between the individual students (for example, it was noted that one of the boys had only recently joined the group and this led us to interpret some of his utterances and other actions as involving bids for inclusion) and between them and the teacher; the communication was multi-modal, including the use of speech, diagrams, gesture, and, at a later stage in the lesson, a calculator display. In order to understand the meanings, in particular the emotional meanings, constructed within this situation, it was necessary to consider the context of culture – the multiple discourses – providing the background of organising concepts structuring the participants' possibilities for meaning making. Here I shall consider just one aspect of this level of context relating to the place of assessment in this classroom and in the broader educational system. In Portugal, students may be judged to fail a year and must then repeat it. This creates positions, defined by explicit criteria external to the students, of *failing student* and successful or *'normal' student*. The schools also use a technology of 'marks' that creates a structure for comparing and ranking students and attaches official positive value to higher rankings. At the same time, however, the researcher's field notes report that in this particular classroom, unlike the more traditional Portuguese classroom, the students "spontaneously and frequently checked their solutions between them, not depending on the teacher evaluation", thus allowing students to adopt the powerful position of evaluator in relation to each other and to their own mathematical work but also, at least in principle, allowing some flexibility with respect to which individual students are able to assert such power at a specific time. Understanding the concepts and values related to evaluation available within the context of culture, including the possibly contradictory nature of some of these, is essential to analysing the possible meanings that the students may have been making when they were involved in specific acts of evaluation of each other and of their work. In the following partial analysis of a brief extract from the lesson it is significant to know that, according to the researcher's background field notes, while the official technology of marks evaluates the three boys as "medium" students with Mário slightly weaker than the others, the boys themselves are said to evaluate each other as “good” (in the case of Filipe and Tiago) and “rather weak” (Mário) students. The field of discourse included in particular the problem on which the students were working and the mathematical resources they were using. The problem, related to locus, involved finding the best position in a field for an irrigation tap; the students were attempting to solve it by using scale drawing and measurement.

(54) Filipe – Quite right! (*Certinho! – subsequent discussion of the translation has suggested that ‘Bang on!’ might be an appropriate colloquial English equivalent*.)

(55) Mário – That’s it! (*É mesmo!*) (*Mário goes with his eyes from his drawing to the eyes of Filipe for a moment and again returns to his drawing*.)

(56) Mário – Quite right! Fantastic! (*Mário turns his eyes again to the eyes of Filipe, he begins smiling, with his right arm touches Filipe in his shoulder for a second*.)

(57) Mário – You know! (*said almost in private to Filipe*)

(58) Filipe – No, it’s a question of doing here to irrigate there for sure, then you try there and, if needed you enlarge it a little (*going with his eyes from his drawing to Mário’s eyes*).

(*Mário is listening to the explanation of Filipe, his eyes in contact to Filipe’s eyes, savouring, delighted, submissive?; he ‘says’ yes with his eyes, agrees with his head; he opens and closes his legs in a movement denoting satisfaction; at this moment Tiago goes from his drawing and looks at Filipe’s drawing*.)

Both Filipe and Mário are making positive evaluations of Filipe’s solution.[4] However, the forms and functions of these evaluations differ, giving rise to different positionings. Filipe’s evaluation appears to relate directly to his concrete solution of the problem. His first utterance, initiating the evaluation sequence, occurs with his successful construction and location of a position for the tap while his second at (58) provides explicit criteria for the evaluation, thus establishing himself both as evaluator and as being in control of the knowledge. Filipe’s position as evaluator in control of the criteria is confirmed repeatedly during the lesson from which this short extract is taken. For example, shortly after this extract he adds to his evaluation by describing the construction as "fitting correctly". Mário, on the other hand, does not indicate any criteria, except when echoing Filipe's own words (56), and he attributes the knowledge explicitly to Filipe (57). His repeated endorsements serve to reinforce Filipe's powerful position rather than to claim his own right to evaluate. At the same time, Mário's body language suggests a subordinate position. The interpersonal meanings, including the possibilities for emotional experiences, that may be made by the participants in this episode are structured by the roles that evaluation plays within the context: its importance as a means of establishing rankings and 'normal' or 'failing' student status; the possibilities available in this particular classroom for individual students to claim evaluator status; the pre-existing evaluation of Mário as a 'rather weak' student and as an outsider to the group. Similarly the establishment of Filipe's solution as valid is achieved both by his relatively high status position in the group and by his use of criteria of success related to accurate measurement that are recognised as relevant within the discourses available in this classroom.[5]

The context thus provides the semiotic structure within which exchange of meaning takes place (including in the case above, for example, the concepts and values of the mathematics curriculum and those related to assessment at national and classroom level) but to study meanings within a particular situation also requires tools for examining the communicative exchange itself – the language. There are two fundamental characteristics of Halliday's linguistics: the notions of *system* and *function*. Within a given situation, there is meaning potential associated with the type of situation, constituted by a system of semantic options from which speakers choose. The semantic system or *register* is a realisation of the semiotic structure of the situation type – the "system of meanings that constitutes the 'reality' of the culture" (Halliday, 1978, p.123).[6] It is structured according to the functions that the language (and other systems such as algebraic notation, graphs, etc.) is being used for within the situation. The *ideational* function, that is, the expression of meanings related to the content of the situation, the objects, participant structure, actions and logical relationships between these, is the semantic realisation of the field of discourse. The *interpersonal* function, the expression of meanings related to relationships between the participants and to the identity of the speaker, is the realisation of the tenor of discourse. The *textual* function, the way in which language itself is playing a role within the situation, is the realisation of the mode of discourse. These functions are represented in texts by different parts of the lexico-grammatical system. The relationship between situation type and semantic system allows us, in a very general and non-deterministic way, to predict in both directions. In other words, given a situation, we can predict the types of things that are likely to be said by participants and, conversely, given a text, we can predict the type of situation in which it is likely to have arisen.

The lexico-grammar used to represent the semantic system in texts produced in mathematical situations – the mathematics register – has been characterised by Halliday himself (1974) and elaborated by Pimm (1987), focusing primarily on the characteristics of mathematical language with some attention to numerical and algebraic notation. A problem with this characterisation, as I have suggested above, is the fact that it does not succeed in taking into account variations in the contexts within which mathematical activity takes place. Not only are there major differences in the situation types within which mathematical texts arise (consider, for example, publishing a research article and teaching 7-year-olds), but there are also considerable cultural differences among those who participate in the exchange of mathematical meanings and there are potentially multiple discourses present within a given situation (most mathematics classrooms could serve as examples of this point[7]). Nevertheless, I suggest that most of us would feel quite confident in identifying whether or not a given text, whether from an academic journal or a primary classroom, was 'mathematical' (Morgan, 2001). The metaphor of a *family* of mathematical registers, used by Chapman (1993) to account for the complexity of classroom communication, may provide a useful way of thinking about this issue. We can recognise very different texts as mathematical not because they arise within situations of the same type but because of family resemblances between the situations.

## 3. Methodological tools and fundamental questions

Having introduced some basic concepts, I turn now to consider what adopting a social semiotic perspective means for research in mathematics education and, in particular, what it can offer us in our search to understand mathematical and educational practices. In this section, I shall describe aspects of this approach, outline some of the linguistic tools that I have found most useful, and suggest fundamental questions arising from a social semiotic perspective that can be addressed to communicative exchanges in mathematics education.

The first characteristic of the methodological approach is its focus on text. I am using *text* here to denote any socially coherent piece of language-in-use (where *language* may include or be substituted by other semiotic systems). Thus, a text may be written or spoken, formal or informal, long or short, produced monologically by a single writer/speaker or dialogically by several in interaction. My aim in focusing on texts produced in mathematical situations is not so much to create descriptions of the nature of mathematical language as to provide a means of identifying and interpreting features of the texts that are likely to be of significance to the mathematical and social meanings constructed in the interaction between writers/speakers and readers/listeners. This identification, however, demands descriptive tools in the first instance. The main tools that I use to describe the verbal components of mathematical texts are based on Halliday's systemic functional grammar (1985).[8] Many mathematical texts also contain significant non-verbal components, including algebraic notation, diagrams, tables and graphs. Tools for the description of these components are less fully developed from a systemic functional perspective, though O'Halloran (2003) has made a significant contribution towards this, identifying differences in both grammatical structure and semantic potential between language and mathematical symbolic notation, while Chapman (2003) has adopted a social semiotic approach to analysis of communication in mathematics lessons involving graphical as well as verbal elements. (See also Kress and van Leeuwen (1996, 2001) for an extension of the ideas of systemic functional grammar to non-verbal modes of communication.) However, the examples that I shall be dealing with in this paper do not involve substantial analysis of symbolic or graphical elements so I do not intend to discuss these in detail here.

It is not sufficient merely to describe the features of the text being analysed. Description of the features of mathematical texts in different genres may be useful in itself as a tool for supporting those who are learning to speak and write mathematically, though mathematics has as yet been given relatively little attention in the fields of applied linguistics and English for Specific or Academic Purposes (ESP or EAP). However, what is of primary interest to me is to attempt to interpret the functions that these features fulfil for the participants in the mathematical practices in which the texts are produced and consumed – and hence to gain understanding of the practices themselves.

The notion of function is closely related to that of *choice*. It is by selecting specific textual elements from those available within the linguistic system that particular functions are realised. This selection is both paradigmatic (choosing between substitutable elements) and syntagmatic (choosing how to link the elements into complete texts). Thus, for example, an English lower secondary school textbook (Bullen et al., 2001) contains text A:

> To add and subtract decimals, line up the decimal points. Then work out as for whole numbers.

By substituting some elements, we might form the alternative text B:

To add and subtract decimals, line up the digits in the units column. Then calculate as for whole numbers.

By substituting and changing the way in which the elements are linked, we could form text C:

Decimals are added and subtracted in the same way as whole numbers, first lining up the digits in the units column.

To interpret the effects of these changes, we need to be able to identify the component of the semiotic structure that is realised by each lexicogrammatical choice. Halliday's functional grammar (1985) identifies the following aspects (of course this is only a partial account of the lexicogrammatical features associated with each of the three metafunctional components):

The **ideational** function, realising the field of discourse, is represented in text by choices made within the transitivity system, that is, the types of processes, the participants in those processes and the representation of actors.

The **interpersonal** function, realising the tenor of discourse, is represented in text by the modality: the mood of verbs, the presence or absence of adjuncts and qualifiers that vary the degree of probability or the expression of attitude. It is also affected by the degree of specialism in the register.

The **textual** function, realising the mode of discourse, is represented in text by the thematic structure and the cohesive structures.

In the example above, the first change effected in B is a change in the participants, thus affecting the ideational function. The change from *decimal points* to *digits in the units column* may be interpreted as placing importance on the values of the numbers rather than on the notation. The change in the naming of the process from *work out* to *calculate* is a change from a widely applicable term used in many everyday non-mathematical discourses as well as in mathematics to a more specialised term readily identifiable as mathematical. This change marks the text as a specialist mathematical text and hence the actions of the student following the instructions are constructed as specialist mathematical actions. This affects the interpersonal aspects of the text, changing the positioning of the student-reader and their relationships to the author and subject matter.

Several different kinds of changes have been affected in text C. The use of passive voice *decimals are added and subtracted* followed by the nominalisation *lining up* rather than infinitive *to add and subtract decimals* followed by imperative *line up* obscures the human agency involved. This affects the ideational function, representing mathematical activity as independent of the participation of the human mathematician. Whereas in A and B the reader is addressed directly by the imperative and is expected to play an active role, in C the reader is distanced from the mathematical processes. This affects the interpersonal function, no longer constructing the reader as an active participant. The changes in the ordering of elements of the text affect the textual function. First, the theme of text C, realised by positioning at the beginning of the statement, is *decimals* rather than adding and subtracting. This focuses attention on description of the number systems rather than on procedures for calculation. Further, the prioritising of the comparison with whole numbers over the lining up of the digits highlights the similarities rather than the differences between the two kinds of numbers. Whereas texts A and B present the student with two new pieces of information they have to use in order to add and subtract decimals, text C presents just one new step to be learnt.

Of course, the grammatical analysis by itself is not enough to address questions of interest to mathematics educators such as why the authors of the textbook chose to write text A and which of the texts would be most likely to help a given student learn how to add and subtract decimals (or any other aspect of mathematics). In constructing the analysis above, I have already drawn on some knowledge of a part of the context of culture in order to make sense of differences between the texts. For example, my interpretation of the effect of the thematic structure of text C relies on my knowledge that within mathematics generally and mathematics education in particular there are different meanings and values attached to *number systems* and to *procedures for calculation*. However, the meanings constructed by actual participants can only be interpreted within the contexts in which the interactions of author, text and reader take place. In the case of a passage from a textbook, the meanings constructed by students will be influenced by the practices of their classroom and by their experience of other mathematical texts. It is important to remember that the text itself can only construct an ideal position from which the reader may read it most naturally; this position may be resisted by readers who adopt alternative positions (Kress, 1989). Having said that, however, the texts presented to students as mathematical will contribute to the contextual and linguistic resources that they will bring to make sense of mathematical texts they encounter in the future. Thus, for example, a preponderance of experience of texts that, like text A, thematise procedures may make students more likely to perceive mathematics as consisting of a set of procedures and hence, perhaps, to find it more difficult to engage with relational or logical aspects of the subject. Alternatively, a preponderance of experience with texts like text C, which obscure human agency in mathematics, may contribute to difficulties for some students in seeing themselves as potential mathematicians.

Focussing on the choices provided by the functional system allows us to examine a text produced and consumed in mathematical contexts, identifying how the text might be different and considering the effects of the choices that are realised in the ideational, interpersonal and textual aspects of the text. This approach raises the following questions that I have found particularly relevant in researching within mathematics education:

What is the nature of mathematics and mathematical activity as it is constructed in a text? (*ideational* aspect)

Who does mathematics? Is a human agent present?

What processes are human agents engaged in? For example, do they bring mathematical objects into being (by, for example, defining or imagining), manipulate objects (calculating, measuring), or merely observe?

What kinds of objects are involved in mathematics?

What kind of causal relationships are constructed?

Who are the participants in the interaction (author and reader or speaker(s) and listener(s)) and what relationships do they have to each other and to the subject matter? (*interpersonal* aspect)

To what extent are participants identified as specialists?

Does the author/speaker make claims to authority, to membership of a community, to solidarity with the reader/listener (see Burton and Morgan, 2000)?

What roles are available to the reader/listener? (As mentioned above, it is, of course, possible for readers to resist the roles provided by the text. Such resistance may be visible in multi-vocal texts such as the classroom data presented by Zevenbergen (1998) or Houssart (2001), which show students resisting the roles made available for them by their teachers within the school mathematics culture.)

What role does the text play within the context of situation? For example, does it tell a story, construct a description, give a set of instructions for a calculation, and make an argument? In the case of oral interactions, do these establish a new mathematical concept or procedure, test students' recall or competence, explain a task, develop a proof or a solution to a problem? (*textual* aspect)

The interpretation of answers to these questions and hence of the possible meanings available to participants must of course be made by drawing on knowledge of the contexts of situation and culture. In particular, it is necessary to ask how the constructed image of mathematical activity and the roles of the participants and of the text within it are valued in the various discourses at play in the specific situation. In the next section, I shall illustrate how I have used these questions and tools in investigating students' mathematical writing.

## 4. Investigating students' writing: An example

The examples of writing, I shall look at here, were produced for examination purposes by secondary school students in England. The texts are in the form of reports of mathematical investigative work on a task entitled "Inner Triangles". The specification of the task given to students is included in Appendix I. This "coursework" formed part of the high-stakes GCSE (General Certificate of Secondary Education) examination taken by students at age 16+, was carried out in class and as homework and was assessed by students' own teachers. The first step of the analysis is to describe the contexts of situation and of culture, as understanding the semiotic structure within which a text occurs not only provides the means of interpreting the ways the text may be understood by participants but also focuses analytic attention on aspects of the text that are likely to have significance within the context. In the space available in this paper I cannot give a full description of the context but will highlight a few contextual factors that are particularly significant to the analyses I offer.

The first of these factors is the place of the activity of writing and reading the texts within the formal assessment system. The system structures relationships between student–author, teacher–reader, and the external authority of the examination board, an independent organisation that sets the task, provides criteria and official procedures for the assessment, and controls the quality of teachers' assessments by external moderation of a sample from each school. The outcome of the assessment has important consequences both for students, who need good grades for access to employment and further education opportunities, and for teachers, whose professional standing is affected by the results their students achieve and by their colleagues' and employers' perceptions of their competence as assessors. A second contextual factor is the nature of the discourse surrounding the notion of *investigation* in school mathematics in England. This discourse introduces values related to, among others, exploration, creativity, originality, and the nature of mathematical activity that are at times in tension with the values of the dominant assessment discourse, including reliability and comparability. A fuller analysis of these discourses and the tensions between them may be found in Morgan (1998, especially Chapter 5).

I shall compare extracts from the texts of two students, Steven and Clive (both taught in the same class), focusing primarily on questions about the nature of mathematical activity as it is represented in their texts, though I shall also include some observations on interpersonal aspects of the texts.[9] Choosing an alternative focus or a different selection of analytic tools would clearly highlight different aspects of these texts. In some ways, indeed, the two students have produced very similar texts, presenting inductively generated generalisations with little attempt at justification. This underlying similarity is unsurprising, given the common context in which the two students were working. Even within this common 'investigation' genre, however, the present analysis draws attention to differences between the two texts, suggesting differences in the students' orientation and positioning within the various discourses available to them.

The notions of *pattern* and *generalisation*, in particular generalisation expressed in *formulae*, play important roles both in the immediate context of situation through the instructions given in the statement of the task to "Investigate the relationship . . ." and to "generalise your results" as well as through the assessment criteria (available either directly to the students or mediated by their teacher) and more generally as a part of the broader context of culture through the discourse of investigation in which 'spotting' and generalising patterns is highly valued – though contested (see Hewitt, 1992; Morgan, 1998; Wells, 1993). It is thus of interest to analyse the ways in which the two students present *patterns*, *tables* and *formulae* in their texts and, through this analysis, to see differences in the ways in which their texts construct the nature of mathematical objects and activities. The representation of the nature of mathematics is part of the ideational function of the text and is realised linguistically by the transitivity system. A first step, then, is to look at the objects represented in the text and the processes they are involved in and to identify who or what are the actors in those processes.

In response to the "Inner Triangles" task, both students drew trapezia with various dimensions on isometric paper and constructed tables to record the dimensions and the number of unit triangles for each trapezium. The first student, Steven, used separate tables for trapezia with specific slant lengths. In the extract shown in Figure 1, presented under the heading "PATTENS" (sic), he discussed the patterns he had noticed.

The extensive repetition of lexical items to do with change, difference and, especially, increase (marked in bold in the text) clearly emphasises the importance of these ideas within the field of discourse. It is of interest, however, to go beyond their mere presence in the text to ask how they occur and who or what is the agent of change.

First it is important to note that the word *increase* itself is used to denote both a process (as a verb) and an entity (as a noun). Where Steven presents the process or action of increasing, it is in most cases either without an

I have found that whenever **you increase** the top length or the slant length **the number always goes up** by the same amount (…) This happens when **you adjust** the top length. I have made a table up to show these results on a larger scale.

<u>TABLE TO SHOW DIFFERENCE IN UNIT NO. WHEN **TOP LENGTH IS INCREASED**</u>

| | | | | | |
|---|---|---|---|---|---|
| TOP | 1 | 2 | 3 | 4 | 5 |
| BOTTOM | 2 | 3 | 4 | 5 | 6 |
| SLANT | 1 | 1 | 1 | 1 | 1 |
| UNIT No. | 3 | 5 | 7 | 9 | 11 |

As you can see **the unit No. increases** by two every time **the top length increases** by one. This can be done by using any slant No. but if **you change** this you may find that **the unit increases may be** different. e.g.

| | | | | | |
|---|---|---|---|---|---|
| TOP | 1 | 2 | 3 | 4 | 5 |
| BOTTOM | 3 | 4 | 5 | 6 | 7 |
| SLANT | 2 | 2 | 2 | 2 | 2 |
| UNIT No. | 8 | 12 | 16 | 20 | 24 |

This time **the unit increase is** by 4 instead of 2. On the next one when **you increase** the slant to three **it increases** to 6.

| | | | | | |
|---|---|---|---|---|---|
| TOP | 1 | 2 | 3 | 4 | 5 |
| BOTTOM | 3 | 4 | 5 | 6 | 7 |
| SLANT | 3 | 3 | 3 | 3 | 3 |
| UNIT No. | 15 | 21 | 27 | 33 | 39 |

As you can see **the difference is** six. Another interesting pattern is the way in which **the unit No's increase** when the top length stays the same and just **the slant increases**.

| | | | | |
|---|---|---|---|---|
| TOP | 1 | 1 | 1 | 1 |
| BOTTOM | 2 | 3 | 4 | 5 |
| SLANT | 1 | 2 | 3 | 4 |
| UNIT No. | 3 | 8 | 15 | 24 |

**The first increase is** by 5, from 3 to 8 and then from 8 to 15 is 7, and finally **15 to 24 is increased** by 9. This shows that **it increases** by the same amount as before but **increases** by 2. So it would go: 5, 7, 9, 11, … This pattern works whatever the top number is.

*Figure 1*. Extract from Steven's "Inner Triangles" text.

actor at all, through the use of the passive voice (*15 to 24 is increased by 9*), or the length or number itself performs the action intransitively (*the number always goes up* or *the top length increases*). Where this action is explicitly performed transitively by a human agent, it is a general *you* rather than a specific person (*when you increase the slant to three*). Thus, the process of varying the values in the problem is not shown as something done by the author himself; rather, it shifts from being a process that may be carried out by any mathematician (*if you change this* or *when you increase the slant*), to a process performed by mathematical objects themselves (*the unit number increases by two every time the top length increases by one*) or by some unspecified agent (*15 to 24 is increased by 9*), and finally, using the grammatical metaphor of nominalisation, to an object

TABLE I
The human mathematician as manipulator of parameters in Steven's text

| Lines | Human activity | Mathematical outcome |
|---|---|---|
| 1–2 | Whenever **you increase** the top length or the slant length | **The number always goes up** |
| 2 | When **you adjust** the top length | This happens |
| 6–7 | If **you change** this | ***The unit increases may be*** different |
| 8–9 | When **you increase** the slant to three | **It increases** to 6 |



which may itself have properties and variations (*The first increase is by 5*). This nominalisation, by transforming a process into an object, opens up the possibility of a higher complexity of generalisation, taking account of relationships between three variables rather than just two at a time and considering rates of change as well as individual changes, though the ambiguity of reference of *it* and *this* at lines 13–14 suggests that Steven is not completely in control of the language (and perhaps also the mathematics) at this point.

The variation that Steven identifies and describes thus seems to be brought about through the autonomous existence of patterns of relations between numbers rather than directly through human activity. The role of the general mathematician *you* is presented on each occasion as setting the patterns in motion by adjusting the parameters, as illustrated in Table I.

The other aspect of human activity in this text is to observe the patterns. Thus, the author himself is presented as having *found* the pattern and *made a table up to show the results*. Moreover, readers are invited to observe the pattern for themselves (*As you can see*. . .). The positive modality of this address to the reader, of the claim that the pattern is *interesting*, and of the assertion that *The pattern works whatever the top number is* plays an important interpersonal role, building an image of Steven as authoritative (at least in relation to this aspect of his work) and constructing a reader who is expected to be interested in being informed about what Steven has found.

Turning to the second student, Clive, one of the most striking features of his text, illustrated in Figure 2, is the large number of statements declaring the existence of tables and formulae – representations of patterns – and locating them within the text.

Representational objects such as tables, diagrams and formulae clearly play a significant part in mathematics as it is represented in Clive's text while the patterns themselves, so prominent in Steven's text, are

**Below the table shows** the results of a quick conversion table. If you have a trapezium with a slant of 1 and a top of 1 you look on the table and the answer is 3.

Slant

| | | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| | 1 | 3 | 8 | 15 | 24 | 35 |
| T | 2 | 5 | 12 | 21 | 32 | 45 |
| O | 3 | 7 | 16 | 27 | 40 | 55 |
| P | 4 | 9 | 20 | 33 | 48 | 65 |
| | 5 | 11 | 24 | 39 | 56 | 75 |
| | Pattern Gap of | 2 | 4 | 6 | 8 | 10 |

Gap of 2 between each answer

**Here is another quick conversion table**.

| Base | Slant | Top | Total |
|---|---|---|---|
| 2 | 1 | 1 | 3 |
| 4 | 2 | 2 | 12 |
| 6 | 3 | 3 | 27 |
| 8 | 4 | 4 | 48 |
| 10 | 5 | 5 | 75 |

**Below is a formula** that our group work out, **here it is**.

The top + The bottom × The slant

Also **my formula is the one above** but **mine is below**.

(*diagram omitted*)

The numbers can be added together to get the next row of numbers. It can also tell you the answer.

(…)

My formula for the triangle is similar to the trapezium because a triangle is like a trapezium but without a top. **Here it is**

The slant length × The bottom length

**Here is a conversion table for triangles**

(…)

Like the trapeziums and the triangles I found a formula for hexagons quite quickly, **here it is**.

The number of triangles ÷ 3 to give the number of hexagons inside it.

**Here is my conversion table**.

*Figure 2*. Extract from Clive's "Inner Triangles" text.

subordinated. Not only are the representational objects present in the text but also their presence is declared, drawn to the reader's attention by the use of existential and locational statements and often positioned thematically. In some cases these objects are simply declared to exist, independent of agency. In other cases specific human actors are involved as agents in their production (*Below is a formula that our group work out*) or as owners of the objects (*my formula is the one above*). Here there is a difference between tables and diagrams, which are generally presented without human involvement in their construction or ownership, and formulae, which are in each case identified with either the author himself or the group of students with whom he worked. This may mirror the different status of these objects within the context of the assessment criteria. While use of tables, diagrams, and algebraic notation is credited under the heading of "communication", formulae also represent an outcome of the process of generalisation and thus may be seen as results or answers. As I identify below, answers also play an important role in Clive's representation of mathematical activity, so a claim to ownership of these acts to position him as a successful student.

The autonomy of tables and diagrams is further enhanced by their own representation as actors, using verbal processes to inform (*Below the table shows the results of a quick conversion table*). Not only is it the table that shows the results, rather than the author, but the nominal phrase *results of a quick conversion table* suggests that the results arise from the table itself, not from any human activity. Similarly, the diagram *can also tell you the answer*. It is significant that Clive uses *answer* here rather than *number of unit triangles*. The geometrical, numerical and algebraic aspects of the field of discourse are suppressed, substituted by the (discipline-independent) notion of *results* and *answers*, valued by traditional assessment discourse. Similarly, his formulae, which play such a significant part in the text as the products of mathematical activity, do not express relationships between variables but are presented as algorithms for achieving the desired numerical answers: *The number of triangles* $\div$ *3 to give the number of hexagons inside it*. Just as the role of formulae is represented as giving answers, the role of the human mathematician is very different from that seen in Steven's text. Rather than manipulating the parameters of the mathematical situation, Clive's mathematician is primarily concerned with reading the answer from the information provided by the tables and formulae: *If you have a trapezium with a slant of 1 and a top of 1 you look on the table and the answer is 3*.

The focus on answers and the claims to ownership throughout Clive's text serve an interpersonal function, constructing the relationship between the author and his reader as between student and examiner. In displaying his results, there are no suggestions that their mathematical content might be of interest in itself. Moreover, the positive modality of his text serves to present Clive as confident in his work, though explicit statements of confidence, such as *I found a formula for hexagons quite quickly*, are qualified to reduce the modality. In the context of assessment within which this text is situated, such statements could be seen as double edged; on the one hand, the author may be seen as able to solve problems quickly and easily and hence be evaluated highly, while, on the other hand, there is a danger that the author's work might be judged to be trivial because it was too easily completed. Hence, the qualification serves as a hedge to protect the author's 'face' in this situation.

The texts of these two students, both responding to the same problem and both written within the same 'investigation' genre, thus construct different images of the objects of mathematics and the nature of mathematical activity. At the same time they claim different types of authority and construct different 'ideal' positions for their readers. In order to understand the occurrence of these differences between two students taught in the same class and undertaking the same task it is helpful to look again at the context within which they were working, in particular the multiple discourses of the context of culture in which they and their teacher were participating.[10] As I have described above, the practice of mathematical investigation as part of a high-stakes assessment system draws on discourses of investigation and of assessment that involve some contradictory values. This multiplicity in the context provides a semiotic structure that, in spite of the apparently narrow constraints of the production of these texts, allows widely different systems of meanings from which participants may select. Hence, tensions are produced for the participants that are likely to be represented in their texts. In the extracts that I have analysed here, Steven appears to draw primarily on a discourse of investigation, oriented to value exploration of interesting mathematics while Clive draws strongly on an assessment discourse, displaying the 'answers' valued within that discourse. Of course, neither student is entirely consistent throughout his text; I would suggest that each draws to some extent on resources from both investigation and assessment discourses, reflecting the intertwining of the two discourses in the practice of investigative coursework.

Given the place of the task within the assessment system, the question of how the two students' texts are evaluated arises. The teachers responsible for assessing them are also engaging in communicative exchange within essentially the same semiotic structure, although they are likely to have slightly different sets of resources on which to draw. My analyses of interviews with teachers as they engaged with assessing these and other similar student texts (Morgan, 1996, 1998; Morgan et al., 2002b) suggest that tensions are also created for teachers and that these are represented both in variations within the sets of semantic options chosen by individual teachers and in more general differences between teachers as they read, interpret, and evaluate student texts.

## 5. Conclusions: Contribution to mathematics education research

The example I have offered above demonstrates how social semiotics and systemic functional linguistics provide tools that allow a principled description of the language of the texts being studied but also structure interpretation of the functioning of the texts within their contexts of production and consumption. Within the space available here it has been possible to give only a limited glimpse of the variety of situations and issues that might be addressed from this perspective. In particular, the examples of written texts that have been used to illustrate the analytic method involve interaction only at a distance between author and reader, with its associated generic features including greater formality and explicitness. Face-to-face interactions such as those between teacher and students in a classroom situation are likely to have different generic characteristics but can nevertheless be analysed using the same methodological tools. Moreover, the greater complexity of such more interactive situations open up a wider range of possible focuses for analysis. For example, it may be possible to consider the nature of the mathematical objects and activities through analysis of the text as a whole and/or by tracking the contributions of the various individual participants (see Carreira et al., 2002 for such an analysis of a group of students problem solving). The roles and relationships of individuals may also need to considered more dynamically as they are negotiated and develop through the course of the interaction.

The general questions and associated tools identified at the end of Section 3 above can be applied to a number of issues within mathematics education in ways that I believe can both sharpen and enrich research. As an example, research into teachers' and students' beliefs about the nature of mathematics often relies on self-reports and responses to explicit or implicit questioning outside the context of actually doing mathematics. It is notoriously difficult to make connections between the results of such investigations and actual practices of doing or teaching mathematics (see the review by Hoyles (1992) demonstrating the complexity of this research area). Indeed, it can be argued that the results achieved in one context (such as interviewing or answering a questionnaire) offer only tangential evidence of what might be the case in a different context (such as solving a mathematical problem). Analysis of ideational aspects of written or spoken texts produced while doing mathematics, either by individuals or by groups interacting, provides an alternative source of evidence. The example above shows how the analysis has identified major differences in the images of mathematics and of mathematical activity that Steven and Clive have constructed in their texts. The results of such an analysis could complement other methods of investigation. They could also form a basis for addressing further questions about how texts constructing different images of mathematics are produced and read by participants in educational contexts, touching on issues of classroom communication, learning and assessment. For example, what happens when teachers read texts produced by students that construct images of the nature of mathematics at odds with those the teachers might have produced themselves? How and to what extent do students adopt and reproduce the images of the nature of mathematics and of mathematical activity constructed by their teachers in classroom interaction?[11] What effect does resisting the nature of mathematics and mathematical activity constructed by a student's written or oral text have on a teacher's evaluation of the student?

Considering interpersonal aspects of texts produced in mathematics classrooms allows us to consider where power lies and what forms it takes. Tracking the modality of utterances by various participants can provide a systematic means of gaining insight into the dynamics of classroom interactions and the roles of individuals within these. This could contribute towards production of a means of characterising differences and similarities in teaching styles and in student participation.[12] In Morgan et al. (2002a) we use an analysis of claims to power made by students' problem solving in a small group as one of the means of identifying possibilities for emotional experience within the classroom. One part of this analysis was included in Section 2.

The example analysed above identified some contrasting aspects of the identities that Steven and Clive constructed in their texts for themselves and for their readers. In written texts such as these we can only elicit relatively limited pictures of participants' identities. More dialogic texts, produced in face-to-face interactions between two or more participants are likely to provide much more complex data in which the various participants are collaborating and simultaneously vying with each other to establish their own identities and positions in relation to those of others. Again, the lexicogrammatical features that realise the interpersonal functions of language can be used in analysing the production of identities through interaction. This notion of identity is a social rather than a psychological notion in that it concerns the ways in which a participant presents themselves to others through their semantic choices, positions and is positioned by others.

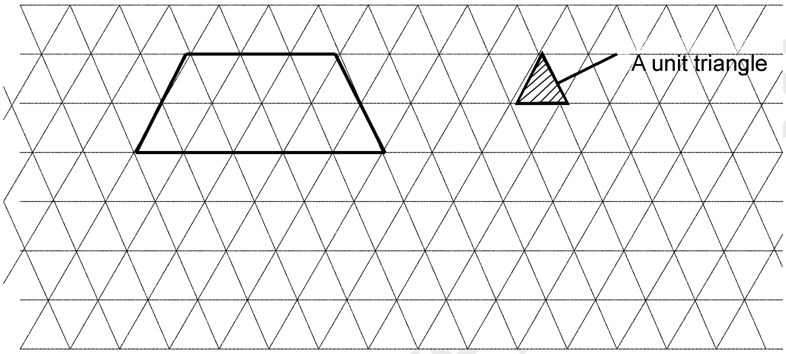
While establishing appropriate identities is of importance to participants in any situation, it is of critical importance to students at all levels whose oral and written productions are to be assessed. It is necessary to establish a degree of authority and confidence that will convince a reader-assessor-teacher without alienating them. The notion of *appropriate* is, of course, dependent on the conventions and power relations of the particular context (Fairclough, 1992). As we found in a study of articles published by research mathematicians (Burton and Morgan, 2000; Morgan, 2003), there is a wide range of ways in which such authors may establish their identities, some of which may be differentially available to participants in different positions within (and on the edges of) the community. At a time when the development of 'authentic' assessment practices in a number of places in the world involves an increase in the extent and complexity of the semiotic resources students need to deploy, I would argue that it is increasingly important to gain knowledge about how various forms are likely to be evaluated. When the student's only choice is between one-letter responses to a multiple-choice item, there are few opportunities for establishing alternative identities. More open and more extended spoken or written responses provide many opportunities – some of which may have negative consequences for students who, perhaps unaware of the interpersonal power of their language, establish themselves as too diffident, over-confident, dependent or arrogant. Greater awareness of the lexico-grammatical choices available within the semantic system and the meanings these may have in specific contexts may help mathematics teachers and students to develop more purposeful and hence more effective use of language.

Halliday's grammatical tools provide systematic means of identifying and describing the choices that authors or speakers have made and the formal impact of these on the ideational, interpersonal or textual functioning of a communicative exchange. I have suggested some areas of mathematics education in which it may be useful to construct such descriptions, together with some illustrative examples. Adopting a social semiotic view of language, however, entails recognising that interpretation of descriptions must always be related to the context of the exchange. This raises two important methodological issues: how much of the context it is necessary to consider and what means to use to describe the context. In the examples I have offered in this paper I have attempted to provide some flavour of the extent of the contexts of situation and of culture taken into account in the analyses and of their use in forming interpretations, though a fuller articulation of social theory is needed in order to characterise the context more systematically.[13]

## Appendix I: specification of the "inner triangles" task

**INNER TRIANGLES**

The diagram below shows a trapezium drawn on triangular lattice or isometric paper.




The trapezium contains 16 of the unit triangles.

The dimensions of this trapezium are

top length 3 units, bottom length 5 units, slant length 2 units.

1. How many unit triangles are there in a trapezium with dimensions
   (a) top length 2 units, bottom length 4 units, slant length 2 units?
   (b) top length 4 units, bottom length 7 units, slant length 3 units?
2. Give the dimensions of a trapezium containing
   (a) 8 unit triangles,
   (b) 32 unit triangles.
3. Investigate the relationship between the dimensions of a trapezium and the number of unit triangles it contains.

*In your report you should*

show all your working, explain your strategies, make use of specific cases, generalise your results, prove or explain any generalisations.

*OPTIONAL EXTENSION*

Extend the investigation in any way you wish.

*For the extension, the only constraints placed on you are that figures must be drawn on isometric paper and that you must look at figures within figures.*

## Notes

1. An exception is Radford's (2003) use of the concept of 'cultural semiotic system' in discussing the development of mathematical thought within the wider context of classical Greek culture.
2. The notions of context of situation and context of culture originated in the work of the anthropologist Malinowski, and have been subsequently elaborated and adapted by linguist Firth and ethnographer Hymes. These notions are discussed by Halliday and Hasan (1989).
3. This analysis was produced as part of the project *Teaching and Learning – Mathematical Thinking*, supported by the Fundação para Ciência e Tecnologia, Grant No. PRAXIS/P/CED/130135/98. The data were originally collected by Madalena Santos, who also provided details of the classroom context and of the Portuguese education system, and an analysis (using a different analytical perspective) is reported by Santos and Matos (1998). I acknowledge the contribution of Madalena Santos and my other colleagues in this project, João Filipe Matos, Susana Carreira, Jeff Evans, Stephen Lerman and Anna Tsatsaroni, to the current analysis (while accepting responsibility for the form in which it is presented here) and am grateful for the enormous contribution that working with them has made to the development of my own thinking.
4. This analysis is adapted from one presented in Morgan et al. (2002a).
5. At a later point in the lesson, an intervention by the teacher introduced a different criterion involving calculation using Pythagoras Theorem. This intervention changed the ways in which the boys were able to make sense of their solutions.
6. The notion of *register*, the semantic system constituting a specific situation type, is also used rather differently to denote the different semantic systems associated with various systems of representation. Thus, Duval (2000) distinguishes between several registers used in mathematics, considering separately natural language, geometrical figures, numeral systems and symbolic or algebraic notations, and graphs. As Duval argues, the meaning potentials of these various registers are different, giving rise to possible difficulties for learners as they attempt to convert representations from one to another. Following Halliday, however, I shall be using *register* in a broader sense, encompassing mathematical meanings realised through any of these systems and combinations of them.
7. See, for example: (Zevenbergen, 1998) for evidence of class-based differences in the meaning potential of classrooms and resistance to the dominant code by working class students; (Carreira et al., 2002) for the use of alternative discourses by members of a group of students as they work together to achieve understanding of the mathematisation of a situation in economics; (Evans, 2000) for analysis of individuals drawing on multiple discourses during problem solving in an interview setting.
8. The specific configuration of tools and interpretations of their significance are addressed to texts in English, though Halliday and others have shown that texts in other languages can be addressed in similar ways (see, for example, Halliday, 1993).
9. This is based on the analysis of these texts presented in Morgan (1995, Appendix 5).
10. It would also be useful to know more about the context of situation within which the texts were produced but adequate data is not available in this case.
11. Chapman (2003) has used a social semiotic approach to address some aspects of this issue in the context of a classroom in which functions are being studied.

12. Atweh et al. (1998) have used social semiotic tools to characterise the differences between lessons by teachers with contrasting pedagogic styles.
13. See Morgan et al., (2002b) for an example of use of Bernstein's social theory (Bernstein, 1996) to characterise the multiple discourses of the context of culture within with teachers read and assessed GCSE coursework texts of the type produced by Steven and Clive.

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